Artificial intelligence for the working mathematician

Lecture 2: LLMs, and what it looks like in practice

Mini-course on emerging applications of AI in theoretical mathematics
ICMAT, Madrid, Spain

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The class of neural networks

Fix a depth n, widths $d = d_0, d_1, \ldots, d_n = m$, and a function $\sigma : \mathbb{R} \to \mathbb{R}$ applied componentwise (for example $\sigma(t) = \max(t, 0)$). For parameters

$$\theta = (W_1, b_1, \ldots, W_n, b_n) \in \mathbb{R}^N, \qquad W_i \in \mathbb{R}^{d_i \times d_{i-1}}, \quad b_i \in \mathbb{R}^{d_i},$$

write $A_i x := W_i x + b_i$ and

$$f_\theta := A_n \circ \sigma \circ A_{n-1} \circ \cdots \circ \sigma \circ A_1 \ : \ \mathbb{R}^d \to \mathbb{R}^m.$$

The *class* is $\mathcal{N} := \{ f_\theta \mid \theta \in \mathbb{R}^N \}$.

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The one structural remark

$\mathcal{N}$ is cut out by finitely many parameters, but it is *not a linear subspace* of $C(\mathbb{R}^d, \mathbb{R}^m)$ — unlike polynomials of degree $\leq k$. Much of the difficulty comes from that.

What the class can represent

Universal approximation (Leshno–Pinkus et al., 1993)

For σ continuous and not a polynomial and $K \subset \mathbb{R}^d$ compact, the *union over all widths* k,

$$\left\{ x \mapsto \sum_{1 \le j \le k} c_j \, \sigma(w_j \cdot x + b_j) \;\middle|\; k \ge 1 \right\}$$

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Do not be impressed

So are polynomials, by Weierstrass. *Density is not the point, and never was*: what separates these families is not what they can approximate, but what happens when you *fit* them.

Training is a regression problem

Given a *sample* $(x_i, y_i)_{1 \leq i \leq M}$ and a *loss* $\ell : \mathbb{R}^m \times \mathbb{R}^m \to [0, \infty[$ — say $\ell(u, v) = |u - v|^2$ — minimise the empirical risk

$$\widehat{R}(\theta) := \frac{1}{M} \sum_{1 \leq i \leq M} \ell(f_\theta(x_i), y_i)$$

over $\theta \in \mathbb{R}^N$. Nobody solves this: one *approximately* optimises it by stochastic gradient methods, the gradient $\nabla_\theta \widehat{R}$ being computed by the chain rule — *backpropagation*.

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So why does that work?

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We are in the situation of thermodynamics before statistical mechanics: it works, it is used, and the explanation is missing.

Stage one: what a language model is

Fix a vocabulary of *tokens* (roughly, word fragments). A language model is a map

$$(t_1, \ldots, t_n) \longmapsto p_\theta(\,\cdot \mid t_1, \ldots, t_n),$$

a distribution over *the next token*; text is generated by sampling, appending and repeating. Pre-training fits θ by maximum likelihood on an enormous corpus, *at a scale no university can afford*.

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The uncomfortable observation

Guess the next word has no business producing anything interesting, yet past a certain scale it does. *The best way to predict the next word is to understand the text.*

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And nobody derived it

These are fitted curves. *There is no argument from first principles saying the exponent should be what it is.*

Stage two: it is not trained the way you use it

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Making it useful is a separate stage — *RLHF*, reinforcement learning from human feedback (Christiano et al., 2017; Ouyang et al., 2022):

(i) humans *write* good answers; fine-tune on them;

(ii) humans *rank* candidate answers; fit a *reward model* r_ϕ to the comparisons;

(iii) optimise the model *against* r_ϕ.

Stage three: replace the human by a checker

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The chain, in one line: predict the next token $\longrightarrow$ satisfy a human judge $\longrightarrow$ satisfy a checker.

Chain of thought: three caveats

It is no longer a prompt. Chain-of-thought prompting (Wei et al., 2022) and "let us think step by step" (Kojima et al., 2022) were *tricks* you applied to a model that would otherwise answer immediately.
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Likewise for the reward: optimise a proxy hard enough and you get what the proxy measures — sycophancy from a human judge, a passed test suite from a checker.

Which machine is which

They are not all "AI", and they do not do the same job.

	Learns	**Searches**	**Certifies**
Computer algebra	no	no	a little
Interval arithmetic	no	no	yes
SAT solver	no	yes	yes
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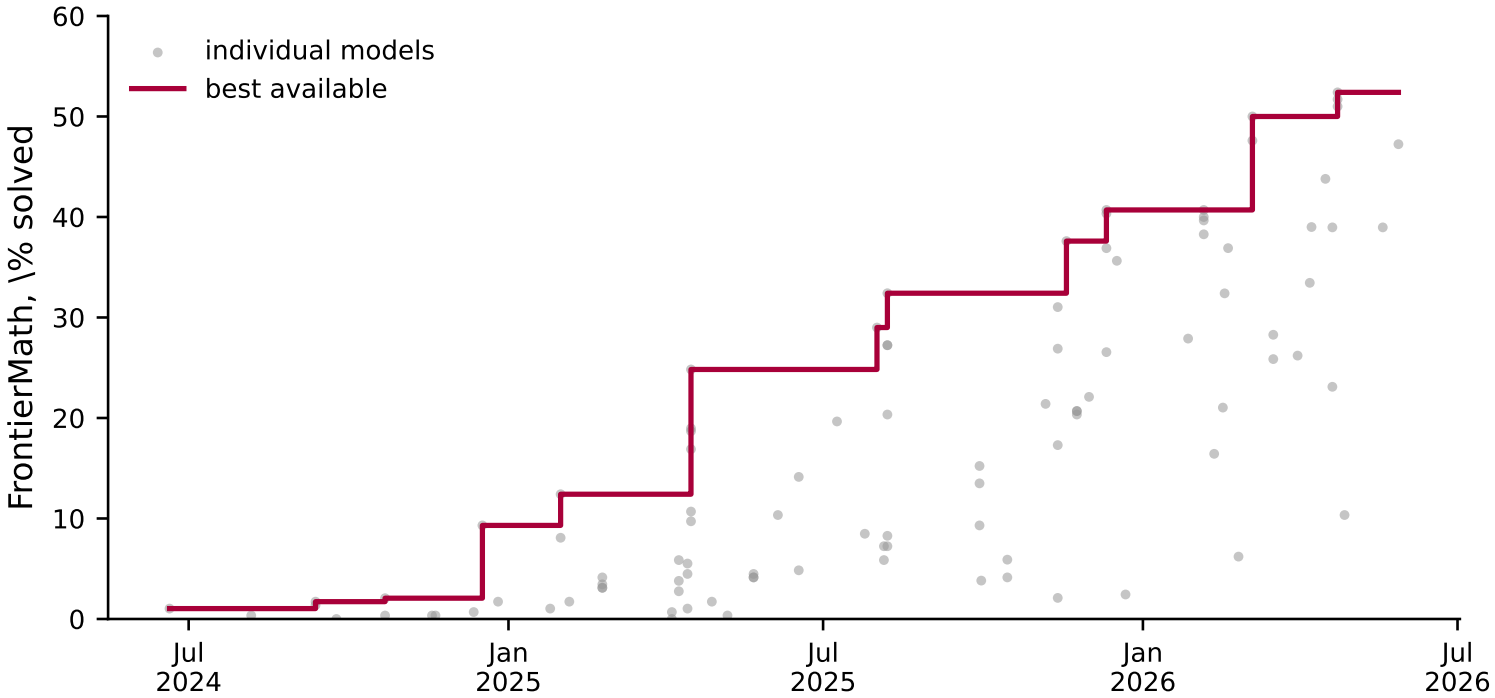
The line that matters

Nothing that learns also certifies. Every certificate in this lecture came from a classical row — which is exactly why an agent's job is to *call* those rows.

One benchmark, over two years



FrontierMath tiers 1–3 (v2): problems written by mathematicians and kept private, solved in a Python-enabled harness. Advanced undergraduate up to early research level — the distinctly research-level problems are tier 4. Data: Epoch AI, *AI Benchmarking Hub* (CC BY 4.0).

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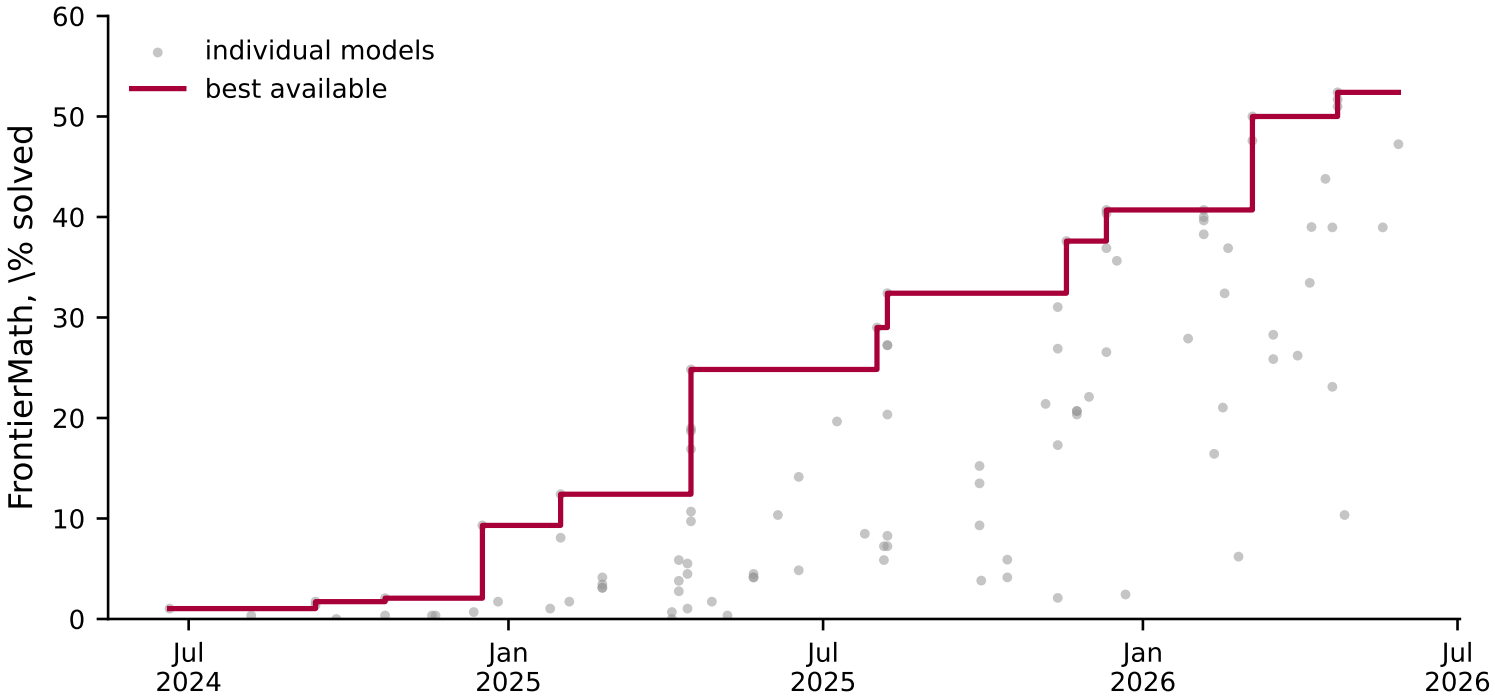

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From 1% in June 2024 to 52% in April 2026. Horizontal axis: release date; each dot is one model–configuration pair.

The IMO as a thermometer

Year	System	Result
2024	AlphaProof + AlphaGeometry 2 (formal input, days of compute)	28/42, silver
2025	OpenAI, Google DeepMind (natural language, in the time limit)	*35/42, gold*
2026†	seven models, independent runs	*42/42*: one first pass

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An opinion formed eighteen months ago is about a different technology.

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Both the hype and the correction are part of the picture. I will come back to the hype on Wednesday.

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The question: among n points in the plane, how many pairs can be at distance exactly 1? Write $U(n)$ for the maximum. Erdős proved $U(n) \geq n^{1+\Omega(1/\log\log n)}$ and conjectured

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What it used, and who checked it

Infinite class field towers of Golod–Shafarevich type: *algebraic number theory, for a question about the plane*. Nine mathematicians — Alon, Bloom, Gowers, Litt, Sawin, Shankar, Tsimerman, Wang, Wood — then published *"a short, digested, human-verified version"*.

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The conjecture: every *polynomial* map $F : \mathbb{C}^n \to \mathbb{C}^n$ whose Jacobian determinant is a nonzero constant has a polynomial inverse. In **July 2026** Alpöge announced a polynomial counterexample in dimension three, found with AI assistance.

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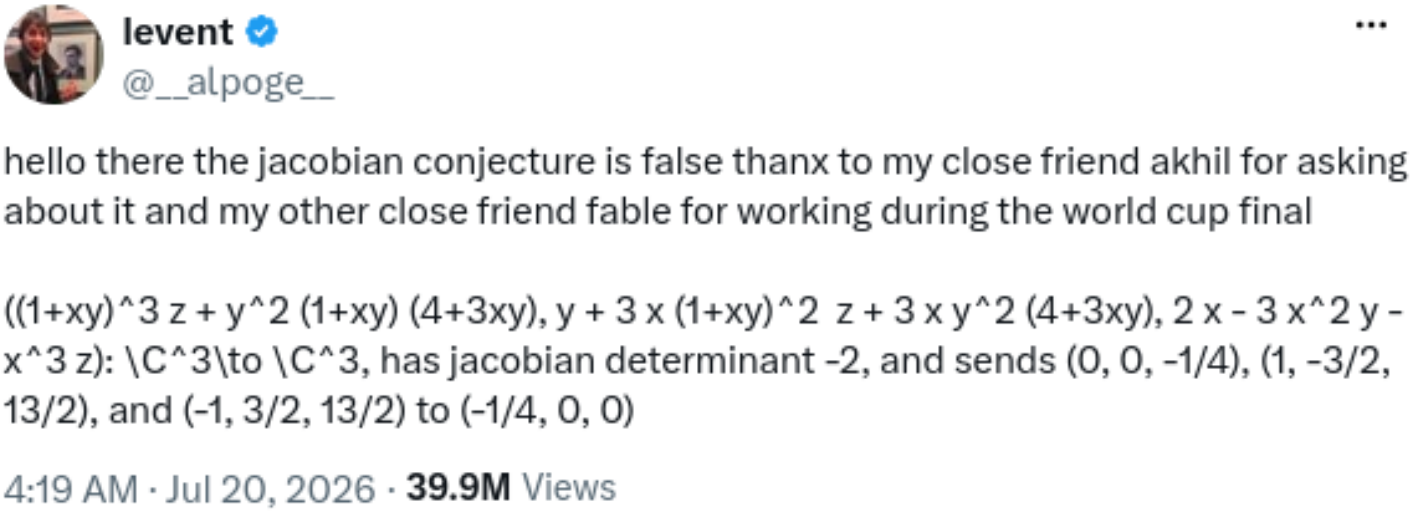

levent
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hello there the jacobian conjecture is false thanx to my close friend akhil for asking about it and my other close friend fable for working during the world cup final

((1+xy)^3 z + y^2 (1+xy) (4+3xy), y + 3 x (1+xy)^2 z + 3 x y^2 (4+3xy), 2 x - 3 x^2 y - x^3 z): \C^3\to \C^3, has jacobian determinant -2, and sends (0, 0, -1/4), (1, -3/2, 13/2), and (-1, 3/2, 13/2) to (-1/4, 0, 0)

4:19 AM · Jul 20, 2026 · **39.9M** Views

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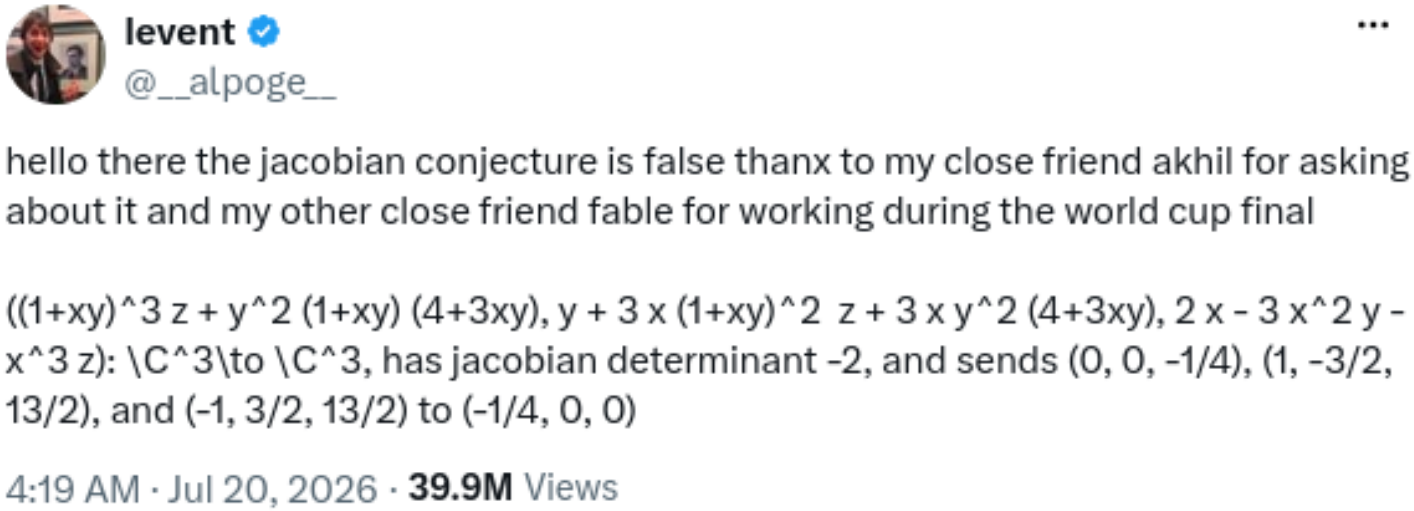

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An infinite family followed the next day (Gallagher), then a geometric explanation (Speyer). False for $n \geq 3$; *$n = 2$ remains open.*

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As a complete account of what these systems do, memorisation is *empirically inadequate*. The paper's other claims are untouched by that.

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- **The data is abundant and machine-readable.** Decades of LaTeX written by careful people — though licensing and quality are uneven.
- **We can manufacture more.** Synthetic problems with known answers, in unlimited quantity — this is exactly how AlphaGeometry was trained.

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- **The data is abundant and machine-readable.** Decades of LaTeX written by careful people — though licensing and quality are uneven.
- **We can manufacture more.** Synthetic problems with known answers, in unlimited quantity — this is exactly how AlphaGeometry was trained.
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Consequence, whether we like it or not

We are not bystanders in this. *Mathematics is one of the main proving grounds*, and the tools are being shaped around what we do.

Six reasons, from my own last twelve months

1. Experiments became cheap.
2. Finding things in the literature became easy.
3. The boring part of the work shrank.
4. You are never stuck alone at 2 a.m.
5. Computation-heavy mathematics became accessible.
6. You can build a tool for *your* problem.

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Not one of these requires you to know anything about machine learning, or to change what you work on.

1–2. Experiments and literature

Experiments

"I wonder what happens to the second eigenvalue if I stretch this edge" used to mean two days of writing throw-away code. *It now means fifteen minutes.*

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Caveat: it also invents references. Always check.

3–4. The boring part, and the 2 a.m. problem

The boring part

Tedious but routine lemmas. Turning a computation into a figure. Rewriting the introduction for the fourth referee.

More time for the real thing. This is not a small effect: measure how much of your week it actually is.

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You will never walk alone

There is now always someone to ask — at midnight, on a Sunday, about a stupid question, without embarrassment.

5–6. Computational mathematics, and tools of your own

The barrier dropped

A rigorous computer-assisted proof used to require a second craft: interval arithmetic, C, parallelism, error analysis.

That barrier is now *much lower*. The mathematics is unchanged — you still have to design the reduction — but the implementation is no longer the reason you abandon the project.

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Do we not all secretly want to turn our research into a video game?

The three shapes of tool

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Rough rule

Chat for *thinking*. Agent for *doing*. Search for *finding an object you cannot guess*.

How to ask: what actually matters

Forget "prompt engineering". Three things matter, and they are the same three that matter when asking a colleague.

1. **Give the context.** Paste the definitions and the paper; say what you tried and why it failed. *The most common mistake is under-specifying.*

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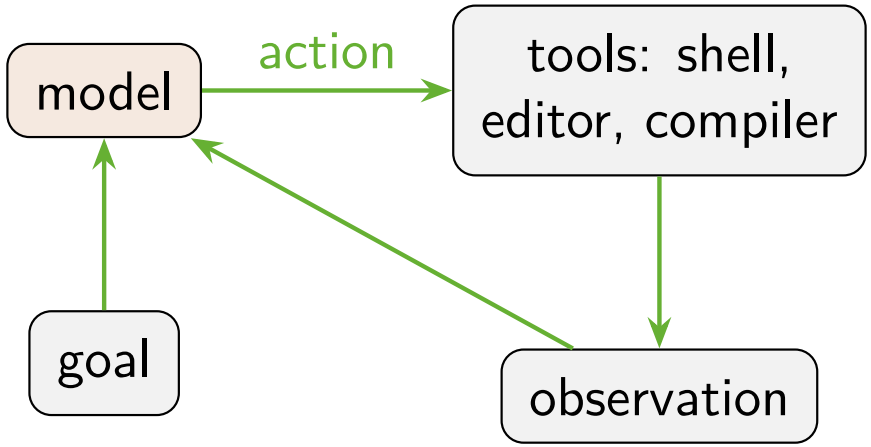
And one rule

Never accept a reference you have not opened. This is where these systems still lie most fluently.

Agents: models with hands

A model alone can only emit text. Give it a loop and some tools:



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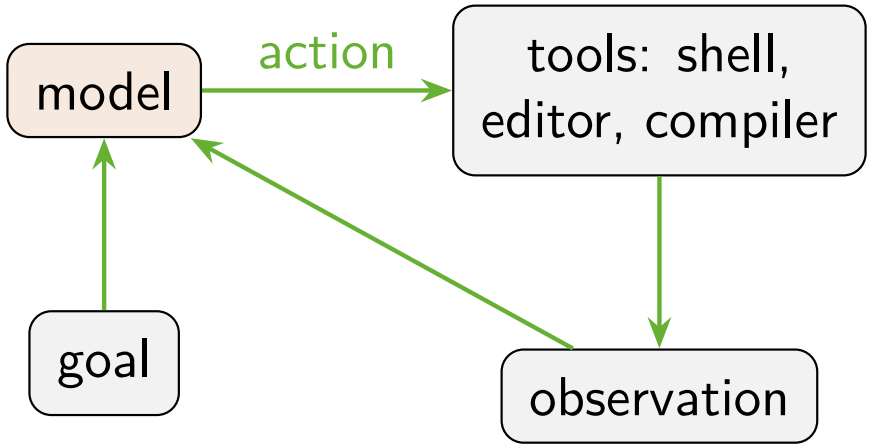

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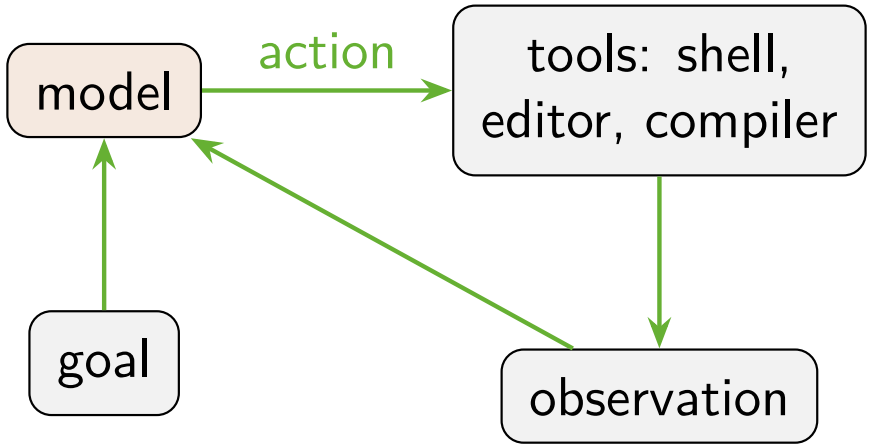

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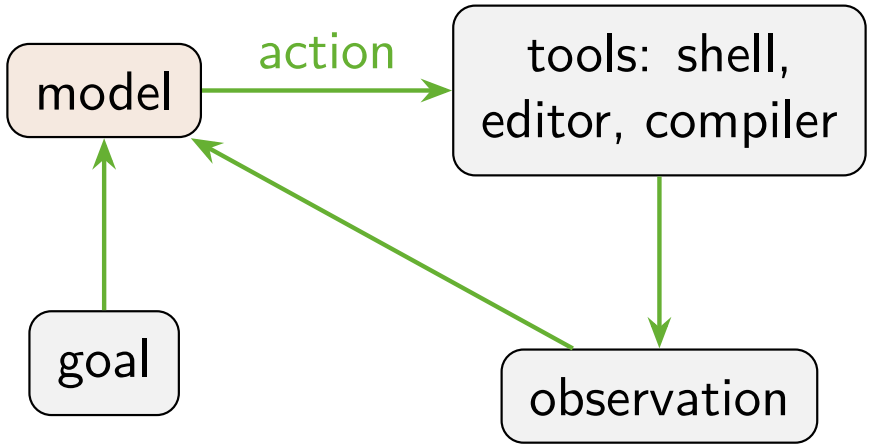

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The rest of today is almost entirely about this.

What an agent changes for us

The loop — write a file, run it, read the error, repeat — runs for minutes or hours. *You are not in it.* You come back to a result.

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Why this matters for us specifically

Technical skill is less of a barrier than it has ever been. What is scarce is knowing *what to compute* and *whether the answer is plausible* — which is exactly what a mathematician has and a programmer does not.

Live demonstration: a tool for one problem

The project

Alvaro Carballeira, Damien Galant, Javier Gómez-Serrano, *The energy of fractional Allen–Cahn layers in dimension one*, arXiv:2608.06226.

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The tool

A dashboard: every claim, its status, its dependencies, and what breaks if a constant changes.

Nobody would fund this. Nobody would write this.

The discipline that makes this safe

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This is exactly the culture the interval arithmetic community built over thirty years. *We should steal it wholesale.*

Finding the conjecture, not the proof

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The network proved nothing. It said which variables mattered.

Learning what a good construction looks like

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PatternBoost found 81. One edge — and the conjecture is false. First progress on it in thirty years.

Evolution over programs

The idea (FunSearch, 2023; AlphaEvolve, 2025)

You do not ask the model for the answer. You ask it for a *program* that constructs a candidate, you *score* the candidate, and you keep the best ones as inspiration for the next generation.

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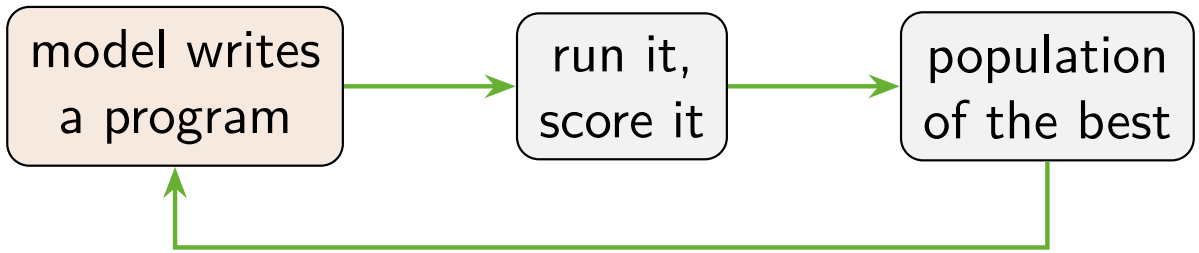

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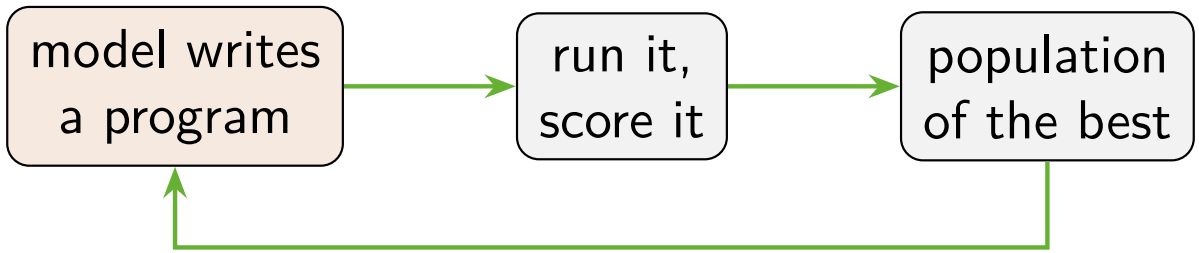

Why this is safe

The output is a *verifiable object* — a construction, an algorithm, a certificate. You check it by hand or by machine. *The model's reasoning is never trusted; only its output is used.*

What it has actually found

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The paper to read

Georgiev, Gómez-Serrano, Tao and Wagner, *Mathematical exploration and discovery at scale*, arXiv:2511.02864 (2025).

Which of your problems this fits

It works when

the object is *finite and explicit* — a configuration, a construction, a set of coefficients; there is a *cheap score*, so you can rank two candidates in milliseconds; and the search space is too large to guess but structured enough that good candidates resemble each other.

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Try this at home

OpenEvolve and *ShinkaEvolve* are free and open source. A first run is an afternoon's work.

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Three steps of one idea: the network *points at the variables* (2021), *proposes the objects* (2024), then *writes the program* that builds them (2025).

What it learned was never mathematics — which variables matter, what a good example looks like, which programs score well. *The mathematics stayed with the mathematician.*

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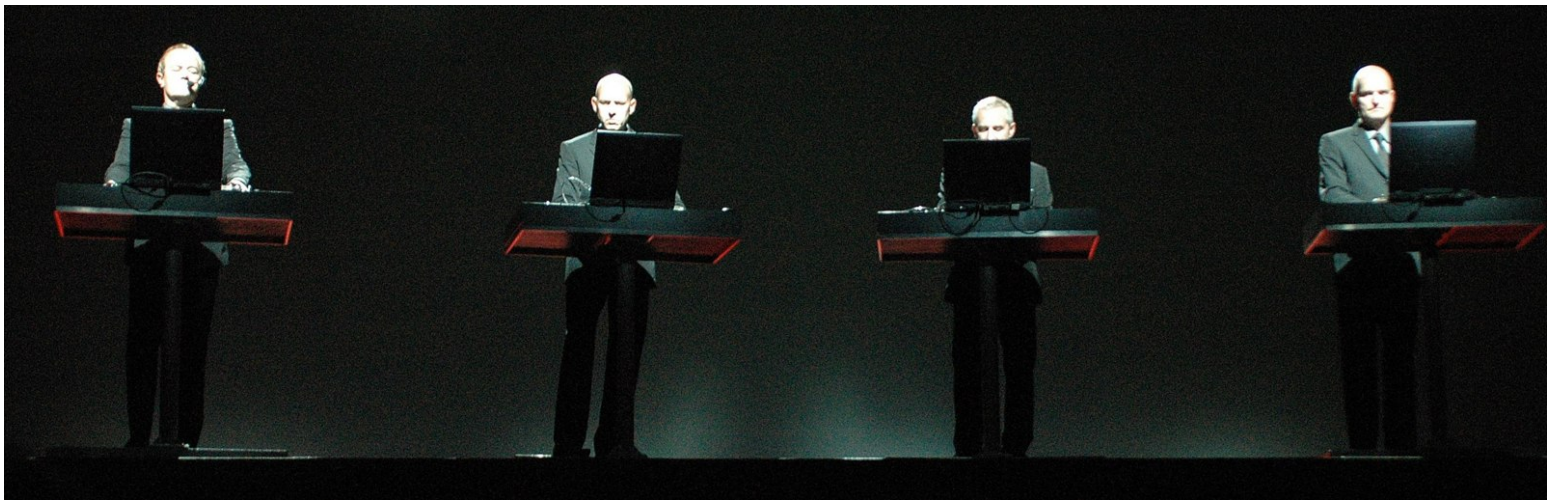
Tomorrow

The uses that have nothing to do with code — and then the difficult conversation: what this does to our community, and what I am genuinely worried about.

¡Hasta mañana!



Kraftwerk, Ferrara, 2005.
Photo Daniele Dalledonne, CC BY-SA 2.0.

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Two conjectures that fell in 2026


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